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Directed-Information Approach for Building Seismic Damage Diagnosis

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Abstract

In this paper, we introduce an information-theoretic approach using building floor vibrations to detect and quantify the building structural damage induced by earthquakes. Previous studies on vibration-based earthquake-induced structural damage diagnosis can be divided into two categories: physics-based methods and data-driven methods. However, the physics-based methods require a lot of prior knowledge about the building structure, which is difficult to be obtained in post-earthquake scenarios. The data-driven approaches detect structural damages using statistical methods to analyze collected structural vibration signals. However, many data-driven methods cannot provide insights into the relationship between extracted features and the physical structural dynamic characteristics. In this paper, we model the process of wave propagation inside building structures as information exchanges and propose an information theoretic approach to extract the representation of information exchanges between the vibration signals on two building floors. We then use the information exchanges as features to detect and quantify the building's structural damage. The advantages of this method are 1) it eliminates the requirement of prior structural knowledge, 2) it allows group analysis of noisy vibration data and provides more detailed information about structural changes, 3) information-theoretic features are physically related to structural damage state, which means the method is not a black-box model and more robust to noise. We evaluate our algorithm using numerical simulation data from 5 buildings under 40 different ground motions. Our method achieves upto 15.48% improvement in damage detection compared to benchmark methods and upto $2.5X$ reduction in the error of damage estimation.

1 Introduction

Accurate and timely building structural damage diagnosis is important to save lives and expedite city reconstruction process in post-earthquake scenarios. Damage diagnosis techniques can help identifying safe shelters to temporally move in, assessing the building safety conditions for evacuation, and determining to rebuild/repair/reserve buildings in an earthquake zone. For example, on the 2011 Tohoku earthquake, there are more than 120,000 buildings destroyed, 278,000 half-destroyed and 726,000 partially destroyed [17]. A fast and accurate inspection of these buildings is critical to accelerate the city reconstruction.

Current practices of building structural damage diagnosis such as manual inspection are mostly labor intensive, time consuming, or error prone. For example, in the Tohoku earthquake, it took many experts more than 1 year to get a full statistics on the overall building damage through visual inspection [17, 28]. Given the drawbacks of current post-earthquake reconnaissance practice, new sensor-based techniques have been actively explored to automate the earthquake-induced building structural damage diagnosis [24, 38].

Recently, people developed vibration-based structural damage diagnosis methods. Based on automatically collected building vibrations during an earthquake, these methods provide the accuracy and speed needed to quickly evaluate the structural health of a building [7, 3, 23, 12]. Most of the vibration-based methods fall into two categories: physics-based methods [2, 8, 34, 35] and data-driven methods [14, 29, 30]. However, the physics-based methods require much prior knowledge about the building structure (e.g. building geometry and material properties), which is difficult to be obtained in the post-earthquake scenario. The data-driven methods utilize statistical models to learn a mapping from the collected vibrations to the structural damage states. Nevertheless, in post-earthquake scenario, the collected building vibration data contains complex environmental noises introduced by the fast-changing seismic dynamics. Moreover, conventional data-driven methods require dense sensor instrumentation to acquire detailed and sufficient information for structural damage detection [21, 22], which is labor-intensive and expensive.

To address these challenges, we introduce an information-theoretic approach to detect and quantify the earthquake-induced building structural damage with sparsely deployed vibration sensors and few prior knowledge about buildings. Our method is based on the premises that wave propagation inside structures can be modeled as the process of information exchanges between adjacent locations, and the structural damage will alter information exchange patterns between two locations. By detecting this change, our method detects and quantifies the damage state of each story inside the building. In this paper, we extract information exchanges using the collected vibration signals at each floor to detect story-level damage states, but the method is generally applicable to any spatial granularity. We extract the information exchanges between the two vibration signals of the floor and the ceiling of each story based on the principles of information theory. With the information exchanges as features of each story, we then estimate the damage state using machine learning techniques. Instead of detecting damages at each sensor location, the presented method detects the damages between sensor pairs, which allows sparsely deployed sensors to infer the structural damages. This method does not require prior knowledge about building. Besides, the bi-directional information exchanges between two collected vibration signals are extracted to provide higher-resolution information about structural properties than conventional correlation-based features [30]. Moreover, we show the analytical relationship between information exchanges and the structural damage of each story to demonstrate that the information exchange is an effective indicator of the structural damage with physical significance.

This paper has 3 key contributions:

1. To best of our knowledge, we are the first to model the wave propagation inside the building as information exchanges as defined in information theory, which allows the analysis of groups of noisy vibration data and provides more detailed information about the structural changes.
2. We present the physical insights of the data-driven information-theoretic approach and the analytical relationship between the information exchanges and the structural properties, which gives theoretical supports to using information exchanges to detect and quantify the structural damage state without prior knowledge of the structure.
3. We evaluate our algorithm using numerical simulation data of multiple buildings with varying heights subjected to multiple earthquake excitations. As a result, our approach achieves up to 15.48% improvement in the damage prediction accuracy.

In this paper, we first provide the physical insight of representing the wave propagation process between adjacent floors as information exchanges in Section 2. In Section 3, we present the analytical relationships between the story-level structural properties and the extracted information exchanges. Then the algorithm of extracting information exchanges as features to detect and quantify the structural damage state is introduced in Section 4. In Section 5, we evaluate our algorithm with data from multiple buildings under a series of earthquake excitations. Finally, Section 6 concludes the paper.

2 Physical Insights Of Information Exchanges Inside Structures

This section provides the physical insights of representing the seismic wave propagation as a process of information exchanges between floors inside a building. When the vibration wave propagates from the floor to the ceiling of one story, the waveform is distorted due to energy dissipation inside the story [39]. This wave distortion can be represented by information exchange. The changes of information exchange reflect the changes of energy dissipation, which depends on the changes of the structural properties. Therefore, by extracting this information, the altering of structural properties can be detected. We can further detect and quantify the structural damages inside the building.

The process of wave propagation inside a building structural system is similar to the process of information exchanges in communication systems [25, 19, 42]. As Figure 1 shows, when earthquake happens, the seismic wave propagates from one location i to the adjacent location j through the structure between the two locations. In this process, there is noise from the non-structural components or other sources which interfere the wave propagation. Similarly, in the communication system, the information is encoded by a transmitter, and sent from the transmitter to the receiver through the channel. In this communication process, the signal may be distorted by the noise when passing through the channel. In structural systems, the structure between location i and j corresponds to the communication channel. When the structure between the locations i and j is damaged, the damage changes the distortion of wave propagation. That is, when the structural damage happens, the difference between information sent at i and received at j, which is also the difference between the vibrations observed at i and j will be different from before damage. The change of the information exchange pattern from i to j indicates the structural damage. In the field of communication system, people developed information theory to study the information exchanges process [25, 19]. Here we model and analyze the wave propagation process in the structural system using information theory. The information exchanges are bi-directional between i and j. When structural property changes, the information exchanges in two directions changes. The changes are different in different directions, which is discussed in Section 3. Therefore, we introduce directed information to quantify the directional information exchanges between two structural response signals.

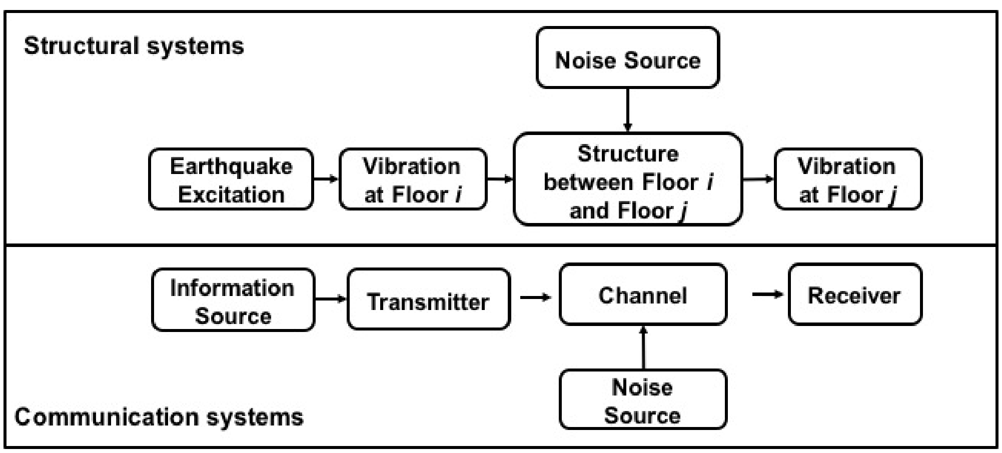


Figure 1: The analogy between wave propagation inside the structural system and information exchanges in the communication system.

When earthquake happens, the seismic wave propagates inside the building. In each story, the seismic waves propagating through the building could be separated into two components: up-going and down-going components. We consider a conceptual model as shown in Figure 2. Whenever the up-going and down-going waves cross a floor interface, they are partly reflected and partly transmitted into the next floor [39]. The transmitted wave would be attenuated along with the propagation path with multiple times of transmissions and reflections. The reflected component would be partly reflected back and partly transmitted in the lower floor and finally attenuate as well. Therefore, the waves observed in the floor and the ceiling are different from each other because the wave is distorted when passing through the structures between them. The wave distortion depends on the properties of the structure that the seismic wave passes by. Meanwhile, the wave distortion between two locations can be represented by information exchanges between two locations' vibrations. Therefore, the structural information is embedded in the information exchanges between the two sensing locations.

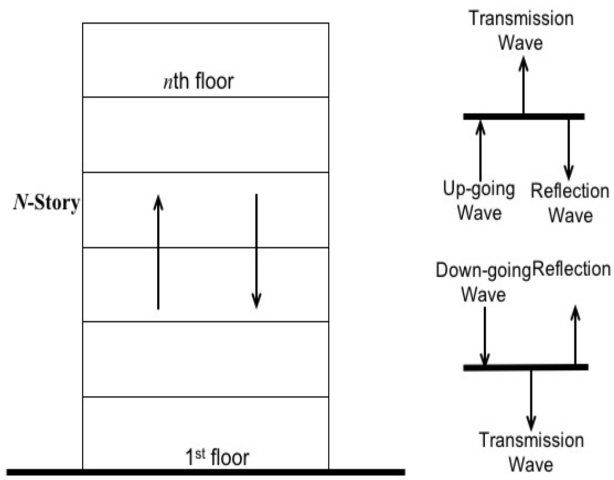


Figure 2: Seismic wave propagation inside the building structure. (a) In a N-story building, the seismic propagate inside the building by pass from one story to the next. (b) The seismic wave propagated inside the building can be decomposed into two parts: up-going component and down-going component. At each interface of floor, the propagated wave will be transmit and reflected. The ratio of transmission and reflection is related to the structure of the floor.

When there is earthquake-induced structural damage in some story, the structural properties of the story change, which also change the information exchanges pattern between the ceiling and the floor vibrations. Since the information about the structure is contained in the wave distortion, the wave distortion between the floor and the ceiling change with different structural damages. For example, given a story, suppose there is a crack appearing in one of the columns during the earthquake. When the seismic wave passes through the column, the energy dissipation becomes different from that in a well-conditioned column due to the crack. Compared to when there is no damage in the column, the wave distortion between the ceiling and the floor carries changes, thus, the information exchange between two the ceiling and the floor vibrations also changes.

By extracting the changes of the information exchanges inside each story, we can detect the changes of the structural properties of the story, and therefore detect and quantify the structural damage state.

3 Analytical interpretation of the relationship between directed information and structural parameters

In this section, we discuss the analytical relationship between the information exchanges and the structural properties. We first introduce the concept of directed information to quantify the information exchanges between two vibration signals. For each story, the directed information is extracted from the vibrations of the floor and the ceiling of the story. We present the physical relationship between the extracted directed information and structural properties of the corresponding story.

3.1 Directed Information

The change of information exchange patterns between the vibration signals at two floors indicates the change of structural properties between the two floors, as Section 2 discussed. Here we use directed information from the field of information theory to quantify the information exchanges between two vibration signals. The information theory is developed to model the information (or uncertainties) contained in random variables (or processes), e.g. seismic wave-induced floor vibrations [43]. Directed information is a concept developed in information theory to measure the directional shared information between two signals.

Directed information is first introduced based on the concept of entropy and mutual information in the field of information theory [13, 25]. In general, entropy quantifies the uncertainty (lack of information) of a random variable. As an example, let random variables X and Y represent the number obtained by tossing a 4-side and 8-side dies, respectively. The entropy of Y will be higher than X, since Y has lower predictability, which means Y has higher uncertainty. When there is dependency relationship between two random variables, the two random variables share part of uncertainties induced by the dependency relationship [3, 37, 6]. Mutual information quantifies this shared uncertainties. This shared information is computed as the information gain (or reduction in uncertainties) for one variable by knowing another related random variable and vice versa [20, 1]. As a natural counterpart, directed information depicts the causal influence that one variable or process (source of information) has on the other variable or process [40, 16, 15]. When there is causal influence between two random variables, directed information is an asymmetric measure that quantifies the shared information with directionality, for example, the information from one random variable to the other random variable. Therefore, compared to the conventional correlation-based features merely focusing on the co-occurrence of two random variables' statistical characteristics, directed information is a more precise measurement providing high-resolution information between two random variables/processes. The concept of directed information has been widely applied in different fields, including identifying the pairwise influence in gene networks [33], neuroscience [41], and stock markets [15].

In our problem, we define two stochastic processes $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ to represent building vibrations at two different floors i and j from the time point t_1 to the time point t_2, respectively. We define the directed information between them using their joint probability density function (PDF). If $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ are independent, their joint distribution is equivalent to the product of their marginal distributions,

$$P(X^i_{t_1:t_2}; X^j_{t_1:t_2}) = P(X^i_{t_1:t_2})P(X^j_{t_1:t_2}). \tag{1}$$

Then, the mutual information of $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ ($I(X^i_{t_1:t_2}; X^j_{t_1:t_2})$) is quantified as the distance (or information discrepancy) between the joint PDF and the product of the marginals by using the concept of Kullback-Leibler divergence (i.e., the mutual information measures the degree of dependency). The distance here represents the information gain when we revise our belief from that $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ are independent to that $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ are dependent:

$$I(X^i_{t_1:t_2}; X^j_{t_1:t_2}) = E[\log \frac{P(X^i_{t_1:t_2}; X^j_{t_1:t_2})}{P(X^i_{t_1:t_2})P(X^j_{t_1:t_2})}]. \tag{2}$$

Mutual information is always non-negative, and it becomes zero when $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ are independent. This mutual information does not represent any directionality in information flow. Hence, an alternative factorization in terms of the joint PDF has been introduced to represent the directionality of information feedforward and feedback between $X^i_{t_1:t_2}$ and $X^j_{t_1:t_2}$ [25]

$$P(X^i_{t_1:t_2}; X^j_{t_1:t_2}) = \overleftarrow{P}(X^i_{t_1:t_2}|X^j_{t_1:t_2})\overrightarrow{P}(X^j_{t_1:t_2}|X^i_{t_1:t_2}), \tag{3}$$

where $\overleftarrow{P}(X^i_{t_1:t_2}|X^j_{t_1:t_2}) = \prod_{t=t_1}^{t_2} P(X^i_{t+1}|X^i_{t_1:t}; X^j_{t_1:t+1})$ and $\overrightarrow{P}(X^j_{t_1:t_2}|X^i_{t_1:t_2}) = \prod_{t=t_1}^{t_2} P(X^j_{t+1}| X^j_{t_1:t}; X^i_{t_1:t+1})$. If we consider $X^i_{t_1:t_2}$ as an input and $X^j_{t_1:t_2}$ as an output, $\overleftarrow{P}(X^i_{t_1:t_2}|X^j_{t_1:t_2})$ and $\overrightarrow{P}(X^j_{t_1:t_2} |X^i_{t_1:t_2})$ correspond to information from j to i and from i to j, respectively.

Similar to the definition of the mutual information where we compare the true joint PDF to the PDF computed as if the processes are independent, the directed information from $X^i_{t_1:t_2}$ to $X^j_{t_1:t_2}$ is defined as the distribution divergence between the true joint distribution and the distribution computed as if $X^i_{t_1:t_2}$ depends on $X^j_{t_1:t_2}$ but not vice versa. When $X^j_{t_1:t_2}$ does not depend on $X^i_{t_1:t_2}$, $\overrightarrow{P}(X^j_{t_1:t_2}|X^i_{t_1:t_2}) = P(X^j_{t_1:t_2})$. Thus, the directed information is defined as:

$$I(X^i_{t_1:t_2} \to X^j_{t_1:t_2}) = E[log\frac{P(X^i_{t_1:t_2}, X^j_{t_1:t_2})}{\overleftarrow{P}(X^i_{t_1:t_2}|X^j_{t_1:t_2})P(X^j_{t_1:t_2})}]. \tag{4}$$

The directed information is smaller than or equivalent to the mutual information. Note that $I(X^i_{t_1:t_2} \to X^j_{t_1:t_2}) \neq I(X^j_{t_1:t_2} \to X^i_{t_1:t_2})$. By the definition of entropy and conditional entropy, the directed information is expressed as follows:

$$I(X^i_{t_1:t_2} \to X^j_{t_1:t_2}) = H(X^j_{t_1:t_2}) - H(X^j_{t_1:t_2}||X^i_{t_1:t_2}), \tag{5}$$

where

$$H(X^j_{t_1:t_2}) = \sum_{t=t_1}^{t_2} H(X^j_{t+1}|X^j_{t_1:t})$$
$$H(X^j_{t_1:t_2}||X^i_{t_1:t_2}) = \sum_{t=t_1}^{t_2} H(X^j_{t+1}|X^j_{t_1:t}, X^i_{t_1:t+1}).$$

The entropy $H(X^j_{t_1:t_2})$ and $H(X^j_{t_1:t_2}||X^i_{t_1:t_2})$ are functionals of the discrete distribution of variables $X^j_{t_1:t_2}$ and $X^j_{t+1}|X^j_{t_1:t}, X^i_{t_1:t+1}$ for $t \in \{t_1, \cdots, t_2\}$. When estimating directed information, we use Equation 5 for computational efficiency, instead of Equation 4 that involves estimating the joint distribution. The entropy values are estimated using the minimax rate-optimal estimators under $l2$ loss [16]. The minimax estimator minimizes the maximum loss function between estimator and functional of real distribution. The loss function is $l2$ norm of difference between estimator and functional of real distribution. We use empirical D-tuple joint distribution based on the collected data to estimate the functionals of real distribution, and it has been proved that empirical joint distribution of D-tuple converges to the true joint distribution [15, 16]. The estimator converges faster and has less mean square error than conventional MLE (Maximum Likelihood Estimator) [15, 16].

3.2 Relationship between Directed Information and Structural Physical Properties

In this section, we show the analytical relationship between the physical properties of building structure and the directed information at each story. The results indicate that directed information extracted between adjacent floor accelerations reflects the structural properties (e.g. stiffness, damping and mass) of the two adjacent two floors, and is a potential damage indicator.

Some assumptions are made to simplify the problem and highlight the important characteristics of the relationship between directed information and structural physical properties. We model the building as a linear multi-degree of freedom system as shown in Figure 3. In the building, the mass is concentrated at each floor. The stiffness of the building is determined by the massless walls and columns.

Given an N-story building, each story is composed of a floor and a ceiling. We collect the earthquake-induced acceleration signal at each floor. We denote the collected acceleration at the nth floor as $\ddot{X}^n$. Similarly, we denote the velocity and relative displacement at the nth floor as $\dot{X}^n$ and X^n, respectively. Given a story n in the building, where $1 \leq n \leq N+1$, the acceleration at the ceiling of the nth story is also the acceleration at the floor of the $(n+1)$th story. We assume the floor acceleration at the base is

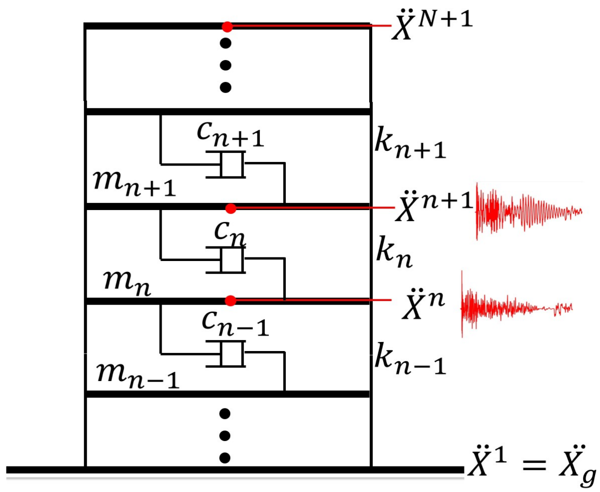


Figure 3: Structural model of a building under the earthquake excitation $\ddot{X}_g$.

the same as the ground motion acceleration, i.e. $\ddot{X}^1 = \ddot{X}_g$, where $\ddot{X}_g$ is the earthquake-induced ground motion acceleration. We denote the mass, shear stiffness, and damping coefficients for the nth story as m_n, k_n and c_n, respectively, as shown in Figure 3.

Denote the mass, stiffness and damping matrices of the building as $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$, and use $\mathbf{X}$ to represent the displacement matrix, i.e. $\mathbf{X} = [X^2, \cdots, X^{N+1}]^T$. The equation of motion is then

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = -\mathbf{M}I\ddot{X}_g, \tag{6}$$

where $\mathbf{I}$ is a vector with all elements as 1. The details of the physical properties matrices are

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & m_N \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & \cdots & 0 \\ -c_2 & c_2 + c_3 & -c_3 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -c_{N-1} & c_{N-1} + c_N & -c_N \\ 0 & \cdots & \cdots & -c_N & c_N \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \cdots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -k_{N-1} & k_{N-1} + k_N & -k_N \\ 0 & \cdots & \cdots & -k_N & k_N \end{bmatrix}$$

Denote $Z = \left[X^2, \dot{X}^2, \cdots, X^{N+1}, \dot{X}^{N+1}\right]^T$ as a multivariate variable. Assume that there exists a zero-mean Gaussian noise V with positive definite covariance matrix Q for floor vibrations. Assume the noises for different stories are independent, i.e. $Q_{2n_1,2n_2} = Q_{2n_1+1,2n_2} = Q_{2n_1+1,2n_2+1} = Q_{2n_1,2n_2+1} = 0, \forall n_1 \neq n_2$ ($Q_{i,j}$ refers to the element in the ith row and jth column of Q). We can transform and discretize the Equation 6, assuming zero-order hold for the ground motion $\ddot{X}_g$, giving

$$Z_{t+1} = A_d Z_t + I^* \ddot{X}_{g,t} + V_d, \tag{7}$$

where each time point represents a sample time of Δt, $V_d \sim N(\mathbf{0}, Q_d)$, and

$$A_d = \exp(A\Delta t); \; Q_d = \int_{\tau=0}^{\tau=\Delta t} \exp(A\tau) Q \exp(A^T\tau) d\tau.$$

For simplicity, we denote the structural responses at the nth floor at the time point of $t+1$ as $\mathbf{X}_{t+1}^n = [X_{t+1}^n, \dot{X}_{t+1}^n]^T$. Equation 7 shows that the structural responses depend on the structural responses of adjacent stories at the previous time point t, i.e., ($\Longleftrightarrow$ refers to the dependency relationship)

$$\cdots \Longleftrightarrow \mathbf{X}_{t:t+1}^{n-1} \Longleftrightarrow \mathbf{X}_{t:t+1}^{n} \Longleftrightarrow \mathbf{X}_{t:t+1}^{n+1} \Longleftrightarrow \cdots . \tag{8}$$

With the dependency relationship described in Equation 8, the dependencies between the structural responses can be described as

$$\mathbf{X}_{1:t}^{n-1} \perp\!\!\!\perp \mathbf{X}_{1:t}^{i} | X_{1:t}^{n}, \forall i > n. \tag{9}$$

Equation 9 represents that given the vibration at the nth floor, the vibration at the lower floor ($< n$) is independent with the vibration at the higher floor ($> n$). Therefore, we have the lemma describing the directed information from $(n+1)$th floor to nth floor at the time point of t is independent of other stories' information:

Lemma 1 *The directed information at time point of t from the $n+1$th floor to nth floor is independent with the information from other nonadjacent floors, i.e.*

$$\begin{aligned} &H(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n) - H(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t+1}^{n+1}) \\ &= H(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1}) - H(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t+1}^{n+1}, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1}). \end{aligned}$$

With Lemma 1, we can obtain the directed information from $(n+1)$th floor to nth floor using structural properties and the white noises. Given $\ddot{X}_{g,1:t}$, $V_d \sim N(\mathbf{0}, Q_d)$, and the starting states $\mathbf{X}_0^n, \forall n$, since linear transform of a Gaussian variable is still Gaussian variable, $\mathbf{X}_t^{n+1}$ subjects to a Gaussian distribution. Denote the $(2n-1)$th and $2n$th rows of the matrix A_d as $A_d(2n-1:2n, \cdot) = [A_d^1, \cdots, A_d^N]$, where A_d^n has size of 2×2. Similarly, denote $M_d(2n-1:2n, \cdot) = [M_d^1, \cdots, M_d^N]$ and $I^*(2n-1:2n, \cdot) = [I_n^*, \cdots, I_N^*]$. Therefore, the conditional variables are expressed as

$$\begin{aligned} &\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1} \\ &= A_d^n \mathbf{X}_t^n + A_d^{n-1} \mathbf{X}_t^{n-1} + \left(A_d^{n+1}\right)^t \mathbf{X}_0^{n+1} + f_1(A_d^n, A_d^{n+1}, A_d^{n+2}, \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t}^{n+2}) \\ &+ \sum_{j=1}^{t-1} \left(A_d^{n+1}\right)^j V_d^{n+1} + I_n^* \ddot{X}_g + V_d^n, \\ &\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t+1}^{n+1}, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1} = A_d^n \mathbf{X}_t^n + A_d^{n-1} \mathbf{X}_t^{n-1} + A_d^{n+1} \mathbf{X}_t^{n+1} + I_n^* \ddot{X}_g + V_d^n. \end{aligned}$$

where f_1 is an implicit function involving with the influence of the structural vibrations at nth and $n+2$th floor. A_d^n can be approximated as follows by Euler's method:

$$A_d^n \approx \begin{bmatrix} 1 & \Delta t \\ -\dfrac{k_n + k_{n+1}}{m_n} \Delta t & 1 - \dfrac{c_n + c_{n+1}}{m_n} \end{bmatrix}^T .$$

The variance matrices for the two conditional distributions are

$$Var(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1}) = Var\left[\sum_{j=1}^{t-1} \left(A_d^{n+1}\right)^j V_d^{n+1}\right] + Var(V_d^n) \tag{10}$$

$$Var(\mathbf{X}_{t+1}^n | \mathbf{X}_{1:t}^n, \mathbf{X}_{1:t+1}^{n+1}, \mathbf{X}_{1:t}^{n+2}, \mathbf{X}_{1:t}^{n-1}) = Var(V_d^n) = Q_d^n, \tag{11}$$

where Q_d^n refers to the covariance matrix for variable $\mathbf{X}^n = [X_t^n, \dot{X}_t^n]$, which is obtained from Equation 7. Since it is assumed that the process noise for different floor responses are independent, we can obtain

$$Q_d^n = \int_{\tau=0}^{\tau=\Delta t} A_d^{n,n} Q^n (A_d^{n,n})^T d\tau + \mathcal{O}(\Delta t^3),$$

where

$$A_d^{j,n} = \begin{bmatrix} A_d(2n-1, 2j-1) & A_d(2n-1, 2j-1) \\ A_d(2n, 2j) & A_d(2n, 2j) \end{bmatrix}, Q^n = \begin{bmatrix} Cov(\dot{X}^n, \dot{X}^n) & Cov(\ddot{X}^n, \dot{X}^n) \\ Cov(\dot{X}^n, \ddot{X}^n) & Cov(\ddot{X}^n, \ddot{X}^n) \end{bmatrix}.$$

Let $P_t^{n+1} = Var\left[\sum_{j=1}^{t-1} \left(A_d^{n+1}\right)^j V_d^{n+1}\right]$. P_t^{n+1} depends and only depends on the structural properties of the floor and the ceiling of the $(n+1)$th story and the Gaussian noise on the structural responses of the ceiling of the nth story.

The entropy of multivariate Gaussian distribution with variance matrix of Q is

$$\frac{1}{2} ln \text{det}(2\pi e Q). \tag{12}$$

Given the definition of directed information in Equation 5, we have the directed information from $(n+1)$th floor to nth floor as

$$I(X_{1:T}^{n+1} \to X_{1:T}^n) = \sum_{t=1}^{T} H(X_{t+1}^n | X_{1:t}^n) - H(X_{t+1}^n | X_{1:t}^n, X_{1:t}^{n+1}) \tag{13}$$

$$= \sum_{t=1}^{T} \frac{1}{2} ln \frac{\text{det}(P_t^{n+1} + Q_d^n)}{\text{det} Q_d^n}. \tag{14}$$

Similarly, we obtain the inverse directed information from nth floor to $(n+1)$th floor as

$$I(X_{1:T}^n \to X_{1:T}^{n+1}) = \sum_{t=1}^{T} H(X_{t+1}^{n+1} | X_{1:t}^{n+1}) - H(X_{t+1}^{n+1} | X_{1:t}^{n+1}, X_{1:t}^n) \tag{15}$$

$$= \sum_{t=1}^{T} \frac{1}{2} ln \frac{\text{det}(P_t^n + Q_d^{n+1})}{\text{det} Q_d^{n+1}}. \tag{16}$$

By the definition of P_t^n, it can be found that the directed information from $(n+1)$th floor to nth floor mainly depends on the structural properties of $(n+1)$th floor and nth floor. If we directly utilize the raw vibration signals, each X^n contains the influence of all the other stories' vibrations and noise during the earthquake. The above proof shows that the directed information effectively helps reduce the noise induced by the structural changes in other inadjacent locations compared to raw vibration signals. Meanwhile, it can be seen that the directed information from the ceiling to the floor of the nth story is different from the directed information in inverse direction. $I(X_{1:T}^n \to X_{1:T}^{n+1})$ focuses more on the properties of the nth floor, while $I(X_{1:T}^{n+1} \to X_{1:T}^n)$ focuses more on the properties of the $(n+1)$th floor. In conventional methods, the information in the two directions is combined and extracted as a feature to infer the changes of structural properties. By differing the information by directionality, the directed information provides more details and enables the analysis of groups of vibration signals.

4 Information-theoretic Approach For Structural Damage Diagnosis

Based on the physical insight and analytical relationship, we propose an information-theoretic approach to detect and quantify the earthquake-induced structural damage using the structural vibration responses to earthquake excitations. In this paper, for simplicity, the approach is explained and implemented for story-level detection and quantification, but the method can be expanded to various scales of detection and quantification, depending on the sensor density. As Figure 4 shows, our approach includes three steps: data collection, feature extraction, and damage modeling. In this section, we first describe the collection of the story-level seismic structural responses and corresponding structural drift ratios. Then we describe how to extract directed and inverse directed information between the accelerations of the floor and ceiling of each story and compute the features. Finally, we train kernel-based support vector machine models for damage detection and damage quantification.

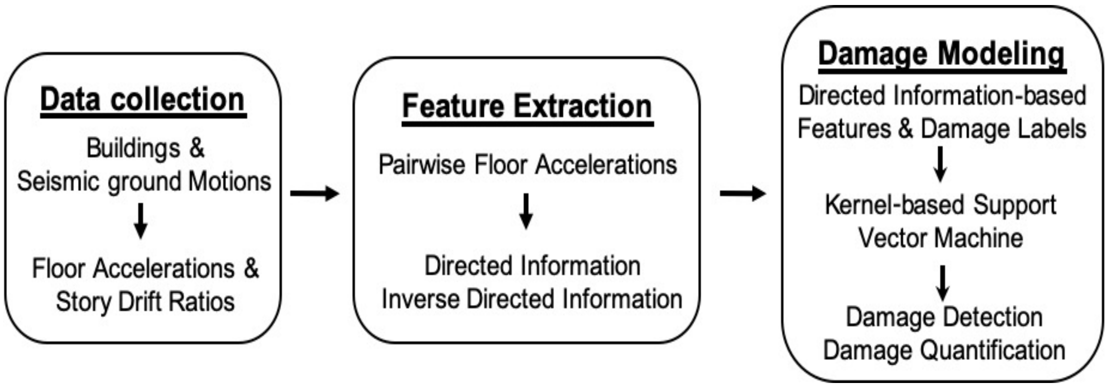


Figure 4: The algorithm overview.

4.1 Data collection

We first collect structural responses to obtain sufficient data samples for damage modeling. In our method, the accelerations at each floor are collected during the earthquake excitation. For example, to extract the directed information as the feature of the nth story, we need the collected vibration signals at nth floor and $(n+1)$th floor. The during-earthquake structural responses depend on the dynamic earthquake excitations and the building structural properties, as shown in Section 3.

After the acceleration data are collected, we use sliding window to separate the vibration signal for further feature extraction. As shown in Section 3, we assume that in a short time window T, the structural properties are consistent. We use a sliding window with the size of T and the stride of 1 to separate the vibration signals into multiple pieces. In this way, each vibration signal is reshaped as a matrix with the size of $(l-T+1)\times T$ matrix, where l is the length of the vibration signal. Then the directed information is extracted from each pair of pre-processed vibration data collected from two adjacent floors.

4.2 Feature extraction

The next step is to extract the directed information from a floor to the ceiling and from a ceiling to the floor as features of the corresponding story. To ensure the computational efficiency, the signals need to be quantized into S level with the principle of $T \approx \dfrac{S^{D+1}}{lnS}$, where T is the short time window, D is the order of the Markov process of seismic-induced vibrations, which is 1 in our scenario based on our state-space model described in Section 3. The directed information is extracted from each pair of sliding-window vibration signals, i.e. $I(X^n_{kT+1:(k+1)T} \to X^{n+1}_{kT+1:(k+1)T})$ and $I(X^{n+1}_{kT+1:(k+1)T} \to X^n_{kT+1:(k+1)T}), \forall k \in \{0, \cdots, K-1\}, n \in \{1, \cdots, N-1\}$. For simplicity, for each story n, we define the directed information from the bottom floor to the ceiling as "directed information" of the nth story, and the directed information from the ceiling to the bottom floor as "inverse directed information" of the

nth story. After computing the directed information and inverse directed information for each sliding window, we obtain the final directed information as well as inverse directed information sequences both with the size of $K \times 1$. The extracted directed information sequences contain the information about how the structural properties change with time evolving.

Given the sliding-windowed vibration signals pair, $X^{n+1}_{kT+1:(k+1)T}$ and $X^{n+1}_{kT+1:(k+1)T}$, we extend the context-tree weighting algorithm [15] to estimate the directed information. As defined in Equation 5, to estimate the directed information $I(X^n_{1:T} \to X^{n+1}_{1:T})$, we separately estimate $H(X^{n+1}_{1:T})$ and $H(X^{n+1}_{1:T}||X^n_{1:T})$. After quantization, the random process becomes discrete. Define $P(X^{n+1}_{t+1}|X^{n+1}_{1:t})$ as the conditional probability mass function for X^{n+1}_{t+1} given $X^{n+1}_{1:t}$, which can be estimated from the vibration signals. We can estimate the entropy as

$$\hat{H}(X^{n+1}_{1:T}) = \frac{1}{T}\sum_{t=1}^{T}\sum_{X^{n+1}_{t+1}} P(X^{n+1}_{t+1}|X^{n+1}_{1:t}) \log \frac{1}{P(X^{n+1}_{t+1}|X^{n+1}_{1:t})} \tag{17}$$

$$\hat{H}(X^{n+1}_{1:T}||X^n_{1:T}) = \frac{1}{T}\sum_{t=1}^{T} f(P(X^n_{t+1}, X^{n+1}_{t+1}|X^n_{1:t}, X^{n+1}_{1:t})), \tag{18}$$

where

$$f(P) = -\sum_{x,y} P(x,y) \log P(y|x).$$

To obtain the entropy and conditional entropy estimators, we employ context-tree weighting algorithm with fixed length of T and context tree depth of D. In this algorithm, we initialize the directed information estimator $\hat{I}(X^n_{1:T} \to X^{n+1}_{1:T})$ as 0. With the quantized sequences $\hat{X}^n_{1:T}$ and $\hat{X}^{n+1}_{1:T}$, we define $Y_t = (X^n_t, X^{n+1}_t), \forall t$. Then $\forall t \in \{D+1, T+1\}$, in the context of $Y_{t-D:t-1}$, we update the context tree for every possible value of Y_t and obtain the estimated probability mass function $P(Y_t|Y_{1:t-1})$. Similarly, we can obtain the estimated probability mass function $P(X^{n+1}_t|X^{n+1}_{1:t-1})$ based on the updated context tree for $X^{n+1}_{t-D:t-1}$. Every update ends with the updating of the directed information estimator

$$\hat{I}(X^n_{1:T} \to X^{n+1}_{1:T}) = \hat{I}(X^n_{1:T} \to X^{n+1}_{1:T}) + f(P(X^n_{t+1}, X^{n+1}_{t+1}|X^n_{1:t}, X^{n+1}_{1:t})) - f(P(X^{n+1}_t|X^{n+1}_{1:t-1})). \tag{19}$$

After iterating for $T - D$ times, we obtain the final directed information by taking the average, i.e. $\hat{I}(X^n_{1:T} \to X^{n+1}_{1:T}) = \dfrac{\hat{I}(X^n_{1:T} \to X^{n+1}_{1:T})}{T-D}$. Similarly, we can estimate the inverse directed information using the same algorithm.

4.3 Damage Detection and Quantification

Given extracted directed information as features, we conduct supervised learning by training different kernel support vector machines for damage detection and quantification. Here, with directed information quantifying the information exchanges between two locations, the task includes two aspects: damage detection and damage quantification. The damage detection focuses on detect whether there exists damage in each story. The damage quantification aims to quantifying the damage severity, which includes classifying the damage into several levels (classification-based quantification) and directly estimating the structural drift between two floors during the earthquake (regression-based quantification).

Story drift ratio (SDR) is a common index for identifying structural damages [31, 4, 32, 18]. We utilize SDR as the ground truth indicator of structural damages. According to FEMA P695 [9], there are five damage states defined for the structure in terms of the peak absolute SDR ($\max(|SDR|)$) at each story, which are no damage ($0\% \le \max(|SDR|) < 1\%$), slight damage ($1\% \le \max(|SDR|) < 2\%$), moderate damage ($2\% \le \max(|SDR|) < 3\%$), severe damage ($3\% \le \max(|SDR|) < 6\%$), and collapse ($6\% \leq \max(|SDR|)$). For the damage detection, according to SDR level, we divide the structural damage state into two classes: damaged ($0\% \leq \max(|SDR|) < 1\%$) and undamaged ($\max(|SDR|) \geq 1\%$). For classification-based quantification, we use the aforementioned 5 classes as the true label. For regression-based damage quantification, we directly use SDR as the true label.

We use kernel support vector machine (SVM) to build the binary-class classification model to detect the structural damage. Given a set of training examples, SVM builds a non-probabilistic binary linear classifier. An SVM model is a mapping from data samples to a new feature representation space so that the examples of the separate classes are divided by a clear gap that is as wide as possible. Kernel support vector machine here is applied for both damage detection and quantification. Kernel support vector machine is good at dealing with the high-dimensional features of data through dimensionality reduction [5, 36]. For damage detection, we use kernel SVM to train the prediction model to detect the damage at each story. For classification-based quantification, we use multi-class kernel SVM. While for regression-based quantification, we use kernel support vector regression to estimate the values of the peak absolute story drift ratio.

5 Evaluation

In this section, we evaluate our approach with both simulated and experimental data. The simulated data are collected from 5 buildings with different heights under 40 earthquake excitations. To evaluate the performance of our features, we compare the performance of the same SVM models trained with our proposed information-theoretic features (*DI-based features*), raw vibration signals (*Signal-based features*), and autoregression coefficients as features (*AR coefficient-based features*), which is another widely used feature for building damage diagnosis [26].

5.1 Data Collection

We collect the structural vibration data at each floor from 5 buildings subjected to 40 earthquake excitations. The archetype of the buildings are located in urban California, United States [10]. The simulation is implemented in an open platform OpenSEES.

There are five archetype steel frame buildings with perimeter steel moment-resisting frames (MRFs). These buildings have 2, 4, 8, 12, and 20 stories respectively, with a first-story height of 4.6m and a typical story height of 4m. More details about archetype buildings' design and geometries are described in the record [27]. The two-dimensional model of each archetype steel building considers the bare structural components of the MRFs. In the analytical model, the steel beams are idealized with an elastic element and a concentrated flexural spring at the center to represent the location of the reduced beam section. Under cyclic loading, the stiffness of steel components and deterioration of flexural strength are captured by modeling the springs with the modified Ibarra-Medina-Krawinkler model. For the first and third mode of all SMFs, Rayleigh damping ratio is assigned with the value of 2%. The natural periods of buildings are recorded.

As specified by FEMA P695, the Far-Field ground motion set is recorded to evaluate the performance of the building models. Horizontal ground-motions are scaled incrementally with respect to the first mode, 5% damped, spectral acceleration $Sa(T_1, 5\%)$ of the steel frame model through collapse. The time histories of floor absolute acceleration and story drift ratio under each incremental ground motion are recorded corresponding to each story of the 5 building models. As an example, Figure 5a and 5b show the story drift ratio of the 1st story and accelerations at the 1st and 2nd floor of a 12-story building under the ground motion observed at the Las Palmas Ave., Glendale station during the 1994 Northridge earthquake.

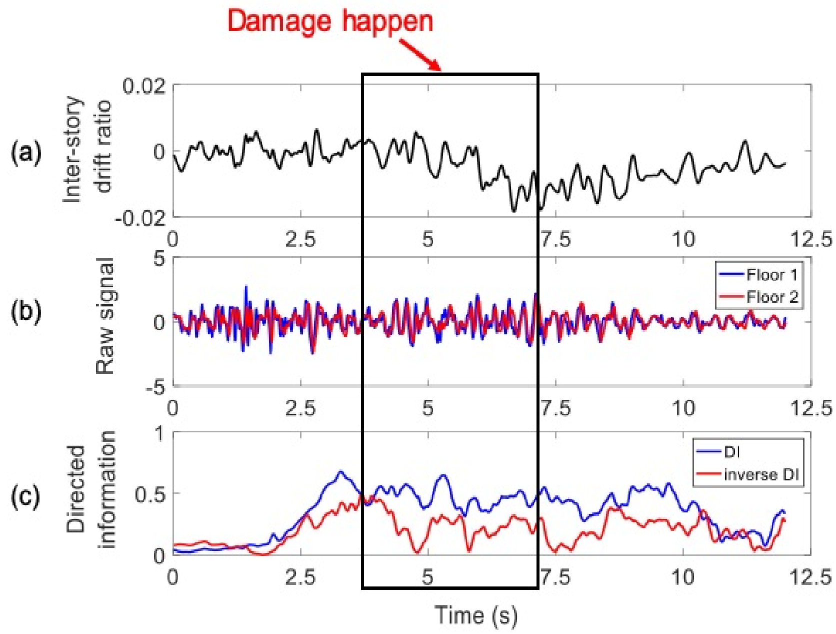
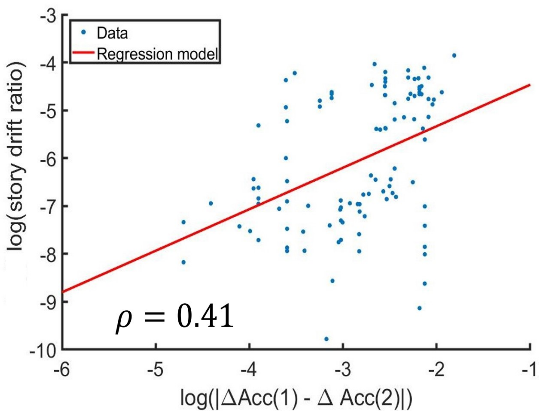


Figure 5: The figures visualize the (a) story drift ratio at the 1st story, (b) raw vibration signals at the 1st floor (blue) and the 2nd floor (red), and (c) directed information from the 1st floor to the 2nd floor (blue), and inverse directed information from the 2st floor to the 1nd floor (red) of a 12-story building under the ground motion observed at the Las Palmas Ave., Glendale station during the 1994 Northridge earthquake. Black box highlights the time interval when damages happen. During damage happening, the inter-story drift ratio decreases to -0.02, and the absolute value of inter-story drift ratio achieves the maximum. From Figure (b), it is difficult to directly distinguish the changes of raw vibration signals induced by structural damages. However, from Figure (c), it is shown that the structural damages induce the difference between information exchange from 1st floor to 2nd floor and from 2nd floor to 1st floor.

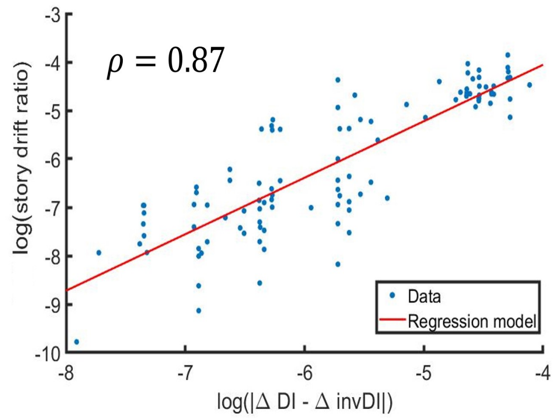
5.2 Feature characterization

We then extract and characterize directed information as effective features to indicate the structural damages. To extract the directed information, we first quantize the vibration signals. In our case, the signals are quantized into $S = 10$ levels. In general, to effectively estimate the directed information, a large number of quantization level is desirable. This is because with large quantization levels, the signal amplitude range for each quantization level is small (i.e., higher signal resolution) such that more information contained in signals can be extracted. However, this level cannot be too large because of the limitation of $T^* \approx \frac{S^{D+1}}{lnS}$. T^* is the sufficient sample number for calculating directed information between two signals. The sufficient sample number needs to be guaranteed to lower the estimation risk, i.e. $T > T^*$, where T is the final sliding window size we select. With the quantized vibration signals, we obtain the directed information and inverse directed information. By aligning the directed information and inverse directed information, we obtain the feature for each sample as a vector with length of $2(l - T + 1)$.

As an example, Figure 5c shows the extracted directed information and inverse directed information between the 1st floor and the 2nd floor of a 12-story building under a ground motion observed from the Northridge earthquake. We can find that between 3.75s and 7s, the absolute story drift ratio increases significantly and a more severe damage happens to the 1st story, as shown in Figure 5a. As shown in Figure 5b, it is difficult to observe the changes by comparing the accelerations collected at the bottom floor and the ceiling. However, from Figure 5c, we observe that at the time of 3.75s, the difference between the directed information and inverse directed information suddenly increases and exhibit different trends. This shows that directed information is an effective damage index for the structural health conditions.



(a)



(b)

Figure 6: (a) The logarithmic correlation between the differences between the gradients of accelerations at adjacent floors and the corresponding peak absolute story drift ratios in a 12-story building. (b) The logarithmic correlation between the differences between the gradients of directed information and inverse directed information on each story and the corresponding peak absolute story drift ratios in a 12-story building. ρ represents the correlation coefficient between features and the absolute inter-story drift ratio. Compared the results in (a) and (b), the directed information-based feature shows a tighter positive linear correlation with the damage states, and thus indicates the potential to be a more predictive features with respect to the structural damages.

To validate the analytical relationship shown in Section 3, we investigate the correlation between the directed information and story drift ratio from the collected data. From the Equations 14 and 16, we can obtain the difference between the gradients of the directed information and inverse directed information as

$$\Delta\mathrm{DI} \triangleq I(X^{n+1}_{1:T} \to X^{n}_{1:T}) - I(X^{n+1}_{1:T-1} \to X^{n}_{1:T-1}) = \frac{1}{2} ln \frac{\det(P^{n+1}_{T} + Q^{n}_{d})}{\det Q^{n}_{d}}.$$

$$\Delta\mathrm{invDI} \triangleq I(X^{n}_{1:T} \to X^{n+1}_{1:T}) - I(X^{n}_{1:T-1} \to X^{n+1}_{1:T-1}) = \frac{1}{2} ln \frac{\det(P^{n}_{T} + Q^{n+1}_{d})}{\det Q^{n+1}_{d}}.$$

$$\Delta\mathrm{DI} - \Delta\mathrm{invDI} = \frac{1}{2} ln \frac{\det(P^{n+1}_{T} + Q^{n}_{d})}{\det(P^{n}_{T} + Q^{n+1}_{d})} \cdot \frac{\det Q^{n+1}_{d}}{\det Q^{n}_{d}}.$$

It is shown that the difference between the gradients is dominated by the structural properties (P^{n}_{T} at the nth story. To explore the correlation between the directed information and the structural damage indicator, i.e., story drift ratio, we plot the pair of peak absolute story drift ratio verses difference of vibrations/directed information for each story in a 12-story building under multiple earthquake ground motions, as shown in Figure 6a. Figure 6a shows the correlation between the difference of the gradient of accelerations at two adjacent floors in each story. The logarithmic correlation between the difference between the gradients of accelerations and the peak absolute story drift ratio is not strong with the correlation coefficient $\rho(\log|\Delta\mathrm{Acc}(1) - \Delta\mathrm{Acc}(2)|, SDR) = 0.41$. In contrast, the differences between the gradients of directed information and inverse directed information have more significant correlation with the story drift ratio with $\rho(\log|\Delta\mathrm{DI} - \Delta\mathrm{invDI}|, SDR) = 0.87$. This is also higher than the correlation coefficient $\rho = 0.69$ of wavelet-based features mentioned in the previous study [11]. Therefore, combining the observations in Figure 5 and Figure 6, we validate our analytical results which show the effectiveness of directed information as a more powerful feature to indicate the structural damages.

5.3 Results and discussion

In this section, we use kernel support vector machine to detect and quantify the structural damage state with the extracted directed information and inverse directed information as features. For the benchmark features based on autoregressive time series modeling of structure's acceleration response, there are several conventional methods for damage sensitive feature extraction. Here, the autoregressive coefficients are extracted by fitting vibration signals in each floor to the autoregressive model, and the coefficients extracted from accelerations in the floor and the ceiling of each story are combined as features for damage estimation. We use the binary-classification accuracy to indicate the damage detection performance, and 5-class classification accuracy to measure the performance of classification-based damage quantification. Meanwhile, to reduce the overfitting of the model, we used cross-validation to calculate the accuracy of the model.

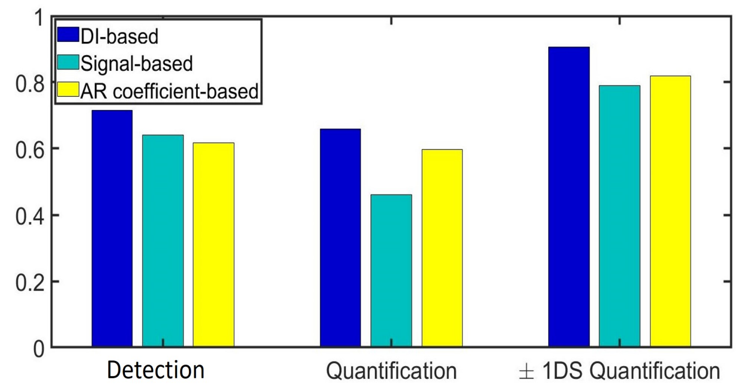


Figure 7: The accuracy of binary damage detection, 5-class damage quantification, and within 1 state damage quantification results using the DI-based feature (blue), signal-based feature (green), and AR coefficient based feature (yellow). The results show that our directed information-based features are more effective to predict the structural damages compared to other methods.

To obtain sufficient data samples, we conduct data augmentation. We use a high-level sliding window with the length of 2000 and the stride of 50 data points to process each vibration signal. The length of the high-level sliding window is decided by the duration it takes for the worst damage to happen from the starting time point. In each high-level sliding window, we extracted the directed information and inverse directed information with a local sliding window length of 200 time points, which is 1 second, as features of each story. Meanwhile, we label each samples according to the peak story drift ratio. If the peak story drift ratio is less than 0.01, we label the sample as undamaged, otherwise we label the sample as damaged. For the damage quantification, we label the sample into 5 classes as introduced in Section 4.3. Finally, we obtain the training dataset collected from the same story at different buildings.

We utilize kernel support vector machine to train the model and utilize cross-validation to evaluate the performance of our approach. With the high-dimensional directed information-based features, the problem of "curse of dimensionality" makes it difficult to optimize the damage model in the original feature space. The kernel trick is applied to reduce the dimensions of feature space to solve it efficiently.

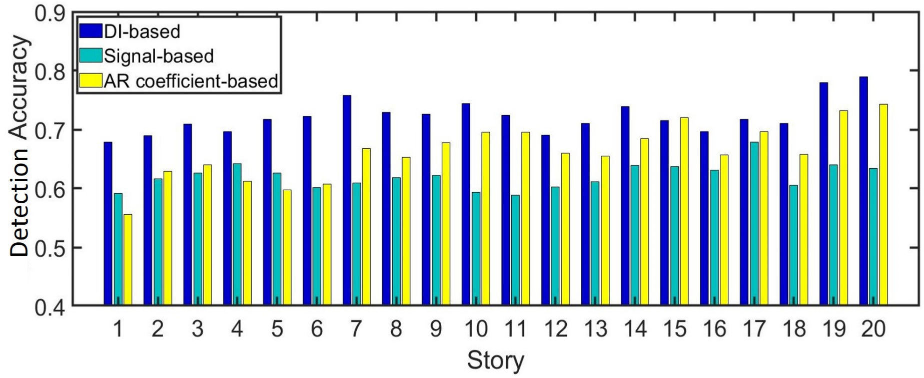


Figure 8: The story level damage detection accuracy using the DI-based features (blue), signal-based features (green), and AR coefficient based features (yellow).

Here we utilize radial basis function kernel-based support vector machine. Both stochastic gradient descent (SGD) and limited-memory Newton algorithms (LBFGS) are applied for computational efficiency. For the bandwidth and coefficient of regularization term, the optimal values 5 and 0.02 are selected respectively using cross-validation. We use 5-fold cross-validation to obtain the final prediction accuracy for different stories. To obtain the overall performance, we take the weighted average across multiple stories according to the corresponding numbers of samples.

Figure 7 shows the damage detection accuracy, damage quantification accuracy, and the damage quantification accuracy within ± 1 damage state with *DI-based features*, *Signal-based features*, and *AR coefficient-based features*. For our *DI-based features*, the damage detection, damage quantification and ± 1 damage quantification accuracy are 71.49%, 65.96% and 90.59%, respectively. It outperforms other features on all 3 types of tasks. Compared to the conventional features, our information-theoretic approach achieve upto 9.8% improvement in the damage detection and 6.27% in the 5-class damage quantification. The accuracy of damage detection is higher than damage quantification, showing that the difference between damaged state and undamaged state is easier to learn by the model than the difference among 5 types of damage severity. As for the damage quantification accuracy within 1 state error, our *DI-based features* also achieves higher accuracy than other features, which shows that our feature is more effective and informative to indicate the underlying structural damage patterns.

Figure 8 presents the damage detection accuracy at different stories. Our DI-based feature achieves higher damage detection accuracy compared to the signal-based method and AR coefficient-based method except at story 15. Especially, at the 20th story, our method achieves 15.48% improvement compared to the signal-based method. In lower stories ($1 \sim 4$), the damage detection accuracy tends to be low. This may be because we combine the data from different buildings under different ground motions to train and test. However, different buildings may have different damage patterns under various earthquakes. For higher stories ($19 \sim 20$), the data is only collected from the 20-story building. Therefore, the training and test data have similar distributions for higher story compared to lower story, which is one reason that higher story has higher accuracy. Another other possible reason is that, the noise would attenuate when propagated to the higher stories, which makes extracted features more effective for damage prediction. Although the general accuracy tends to be low in lower stories, our method has the most significant improvements in accuracy in these levels, as shown in Figure 8.

6 Conclusion

This paper presents a new information-theoretic approach to diagnosing earthquake-induced structural damage. In our method, the process of wave propagation inside the building structure system is modeled as the process of information exchanges. We show both the physical insight and analytical proof of the physical relationship between structural dynamic characteristics and information exchanges. Extracted from structural vibration signals at each floor, the information exchanges are used as features for damage detection and quantification in story-level.

Our information-theoretic approach is evaluated in both simulated structural vibration data and experimental structural vibration data. As a result, our approach achieves upto 15.48% improvement in damage detection compared to benchmark methods and upto 90.59% accuracy on ± 1 damage quantification. This information-theoretic approach does not need to assume a particular structural model, or probability distribution of the vibration data. Furthermore, our approach uses only during-earthquake data from sparsely deployed sensors for detecting the existence of damage and estimating the actual story drift ratio at each story in a computationally efficient way.

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